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Department of Electrical and Computer Engineering School of Engineering

Final Report
February 15, 1994 - September 30, 1995
Contract No. N00014-94-I-0461

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# OPTIMAL PHYSICS-BASED DETECTION AND CLASSIFICATION OF OBJECTS IN THE VICINITY OF AN UNCERTAIN SEAFLOOR

Approved:

L. W. Nolte, Principal Investigator

Prepared under: Office of Naval Research (Code 1125OA)

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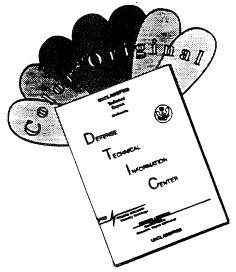
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## 1 Introduction

The objective is the design of a robust algorithm for the active detection of an unknown object suspended in the water column in the vicinity of the seafloor. The optimum detection of an unknown object in an uncertain random wave scattering environment is considered. The spatial variability and lack of precise knowledge concerning the properties of the ocean acoustic propagation medium have prompted the investigation of optimum signal processing techniques for source localization and parameter estimation that are robust with respect to various forms of environmental uncertainty. The principal source of reverberation is acoustic interaction with a randomly rough anisotropic ocean bottom with imperfectly known correlation structure. A physics-based approach to the design of the optimum detector is presented which merges statistical physical modeling of the acoustic scattering medium with a probabilistic description of environmental prior knowledge within a Bayesian decision-theoretic framework.

# 2 Approach

A physics-based approach to the design of the optimum detector is presented which merges statistical physical modeling of the acoustic scattering medium with a probabilistic description of environmental prior knowledge within a Bayesian decision-theoretic framework. For the high-frequency, shallow water, reverberation-limited environment considered herein, the parameterization of the acoustic medium is essentially limited to modeling acoustic interaction with anisotropic seafloor microroughness with unknown horizontal wave-number spectrum parameters. Simulation results, presented in terms of receiver operator characteristic (ROC) curves, aim to illustrate three principal points: (1) the cost of ignoring the bottom reverberation spatial coherence when it is present in the data; (2) the sensitivity of the likelihood ratio detector for a known environment to incorrect prior knowledge of the microroughness wave-number spectrum; and (3) the robust performance realizable by the optimum detection algorithm that properly accounts for environmental uncertainty within a Bayesian framework.

# 3 Summary of Research

The research is summarized in the enclosed technical paper, V. Premus, D. Alexandrou, and L. W. Nolte, "Full-field optimum detection in an uncertain, anisotropic random wave scattering environment", *J. Acoust. Soc. Am.*, **98** (2), Pt. 1, pp. 1097-1110, August, 1995.

# 4 Participating Scientific Personnel

- L. W. Nolte, Principal Investigator, Professor of Electrical Engineering, Duke University
- Dimitri Alexandrou, Co-principal investigator, Associate Professor of Electrical Engineering, Duke University.
- V. Premus, "Optimal physics-based signal processingin a random wave scattering environment", Ph.D. Thesis, Duke University, November 1994.

# 5 Publications and Papers

- V. Premus, D. Alexandrou, and L. W. Nolte, "Full-field optimum detection in an uncertain, anisotropic random wave scattering environment", *J. Acoust. Soc. Am.*, 98 (2), Pt. 1, pp. 1097-1110, August, 1995.
- V. Premus, D. Alexandrou, and L. W. Nolte, "A generalized framework for high-frequency optimum target detection in a shallow water, random wave scattering environment", *Meeting of the Acoustical Society*, Austin, Texas, December 1994.
- V. Premus, "Optimal physics-based signal processingin a random wave scattering environment", Ph.D. Thesis, Duke University, November 1994.

# Full-field optimum detection in an uncertain, anisotropic random wave scattering environment

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The optimum detection of an unknown object in an uncertain random wave scattering environment is considered. A physics-based approach to the design of the optimum detector is presented which merges statistical physical modeling of the acoustic scattering medium with a probabilistic description of environmental prior knowledge within a Bayesian decision-theoretic framework. For the high-frequency, shallow water, reverberation-limited environment considered herein, the parametrization of the acoustic medium is essentially limited to modeling acoustic interaction with anisotropic seafloor microroughness with unknown horizontal wave-number spectrum parameters. Simulation results, presented in terms of receiver operation characteristic (ROC) curves, aim to illustrate three principal points: (1) the cost of ignoring the bottom reverberation spatial coherence when it is present in the data; (2) the sensitivity of the likelihood ratio detector for a known environment to incorrect prior knowledge of the microroughness wave-number spectrum; and (3) the robust performance realizable by the optimum detection algorithm that properly accounts for environmental uncertainty within a Bayesian framework. © 1995 Acoustical Society of America.

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#### INTRODUCTION

The spatial variability and lack of precise knowledge concerning the properties of the ocean acoustic propagation medium have prompted the investigation of optimum signal processing techniques for source localization and parameter estimation that are robust with respect to various forms of environmental uncertainty.<sup>1,2</sup> In this paper, the problem of optimum detection of an unknown object in an uncertain reverberation-limited environment is considered. The principal source of reverberation is acoustic interaction with a randomly rough anisotropic ocean bottom with imperfectly known correlation structure. A physics-based approach to the design of the optimum detector is presented which merges statistical physical modeling of the acoustic scattering medium with a probabilistic description of environmental prior knowledge within a Bayesian decision-theoretic framework. A member of a class of optimum signal processing techniques which draws upon accurate physical modeling of randomness in the acoustic medium for the treatment of environmental uncertainty, the approach outlined herein makes use of the probability density function (pdf) of the acoustic field scattered from a randomly rough ocean bottom. A general theoretical framework for deriving the optimum detector is first presented in which the exact analytical form of the scattered field pdf is presumed to be arbitrary or unknown. It is then shown that for the special case of Gaussian reverberation, this general result may be analytically simplified to yield a computationally efficient implementation of the optimum detection algorithm.

The objective is the design of a robust algorithm for the active detection of an unknown object suspended in the water column in the vicinity of the seafloor. For the highfrequency, large grazing incidence configuration adopted herein, the parametrization of the acoustic medium was essentially limited to modeling of acoustic interaction with Goff-Jordan type seafloor microroughness3 with unknown horizontal wave-number spectrum parameters. Intuitively, the detection algorithm seeks to minimize the signal masking property of the bottom reverberation by exploiting the spatial coherence of the backscatter induced by the interface roughness correlation structure. The optimum detector is derived by connecting the statistical description of the random fluctuation in the backscattered acoustic field with a physical model for the reverberation from fine-scale seafloor geomorphology, thus integrating the physics of the acoustic scattering process directly into the formulation of the processor. Further, by working within a Bayesian framework, a mechanism is presented for introducing a probabilistic description of the state of prior knowledge concerning the physical parametrization of the seafloor, i.e., the Goff-Jordan wavenumber spectrum. The broad implication of this work is that optimum signal processing techniques based on the statistical physical modeling of the propagation medium provide a robust mechanism for treating ocean acoustic environmental uncertainty. The approach also illustrates that, notwithstanding the need for more accurate modeling of the target scattering physics, a significant performance gain is realizable based on accurate statistical physical modeling of the medium alone.

The problem of target detection in an oceanic environment in which the principal cause of interference is reverberation has been investigated in the past. For example, Van Trees<sup>4</sup> considered the case of active detection in the presence of volume reverberation. In that work the backscattered field was modeled using a Poisson distributed point scattering

process with probability distributions governing the complex envelope, Doppler shift, and range of each scatterer. The optimum receiver based on this model was derived for the signal-known-exactly (SKE) case and compared to a conventional receiver, e.g., one in which the reverberation component of the acoustic return is completely ignored. It is significant to note that Van Trees emphasized the importance of accounting for the spectral composition of the reverberation field in optimum signal and receiver design. In this paper, we seek to build on this work in two ways. First, we attempt to design an optimum detector that accounts for the spectral composition of bottom reverberation due to acoustic interaction with geologically meaningful bottom roughness. Second, by introducing a Bayesian framework, we seek a robust means for treating the impact of environmental uncertainty on detection performance.

In this paper, it is assumed that the bottom microroughness is the dominant mechanism responsible for acoustic scattering. Consequently, the acoustic scattering model most accurately represents a high-frequency, large-grazing incidence sonar application. It is important to realize, however, that the validity of the optimum detection approach proposed herein is completely general, not inherently bound to any particular acoustic scattering or propagation model, or any particular assumption concerning the pdf of the reverberation. The scattering mechanism emphasized here is selected as one physically meaningful agent for the introduction of randomness into the ocean acoustic medium, in order that the performance of the detection algorithm may be characterized. If, for example, the sonar configuration for a particular application were more accurately characterized by a lowfrequency, near-grazing incidence geometry, with significant penetration of the incident field into the sediment, an acoustic scattering model which accounts for scattering from volume inhomogeneities in the sediment could be substituted.

In Sec. I, the physical model for seafloor microroughness is reviewed. In Sec. II, the acoustic scattering model is presented. It is based on the 3-D Helmholtz-Kirchhoff scattering theory. Physical modeling of the scattered field spatial coherence required for the case of Gaussian reverberation follows the treatment of Restrepo and McDaniel.<sup>5</sup> Representative samples of the reverberation covariance function for different families of surface roughness are presented. In Sec. III, the derivation of the optimum detector in the presence of uncertainty regarding the acoustic signature of the target and wave-number spectrum of the seafloor microroughness is outlined. The special case of Gaussian reverberation is then treated in detail. In Sec. IV, detection performance for the Gaussian case is quantitatively illustrated through the use of receiver operating characteristic (ROC) curves. The simulation results aim to illustrate three main points: (1) the potential cost of ignoring the bottom reverberation spatial coherence when it is present in the observation; (2) the sensitivity of the likelihood ratio detector for a known environment to incorrect prior knowledge of the microroughness wavenumber spectrum; and (3) the robust performance realizable by the optimum detection algorithm that properly accounts for environmental uncertainty within a Bayesian framework. Finally, conclusions are presented in Sec. V.

#### I. THE SEAFLOOR MICROROUGHNESS MODEL

The physical model for surface roughness adopted in this work is due to Goff and Jordan.<sup>3</sup> The Goff-Jordan model is capable of representing two-dimensional, multiscale roughness, with anisotropic correlation properties. As such, it provides a geologically meaningful parametrization of seafloor relief. The model assumes the seafloor to be a stationary, zero mean, Gaussian random field, and thus completely specified by its covariance function. The surface covariance function has five parameters which are used to quantify spatial variability of the topography: rms amplitude H(m), lineation direction  $\zeta_s$ , two characteristic wave numbers  $k_n$  and  $k_s$  in the lineation and cross-lineation directions, respectively, given in cycles/m, and the Hausdorff dimension D.<sup>3</sup>

The Goff-Jordan surface model was initially introduced as a means of stochastically parametrizing the distribution of large-scale (~km) morphological features such as abyssal hills, volcanic cones, etc. The model's extension to the representation of microroughness remains to be definitively established, since the mechanisms responsible for interface roughness at very small and very large scales are quite different. Nevertheless, it has been shown that the proposed extrapolation of the model is capable of simulating a number of naturally occurring classes of microstructure, e.g., sediment ripple fields and isotropic shell hash.<sup>6</sup> As the model presents a convenient means of introducing multiscale anisotropic bottom relief, for the purpose of this work it is assumed that a suitably bandlimited version of the original model is applicable within the range of wave numbers of

It must be emphasized that the detection algorithm in this work operates on observations of the full acoustic field, i.e., the statistical fluctuations of amplitude and phase of the raw reverberation signal, and not on transit time data typically associated with multibeam bathymetric sonar systems. As such, the spatial scale of the roughness to which the detector is sensitive will be dictated by the carrier frequency of the transmitted signal, rather than by the pulse width of the sonar. The statistics of the scattered field amplitude and phase, and hence the performance of the detection algorithm, will be primarily influenced by bottom features on the order of the carrier wavelength. For example, in the case of an active sonar operating at a carrier frequency of 12 kHz, the incident acoustic wavelength is 12.5 cm. Thus the scattered field spatial statistics will be a function of microroughness spectral content approximately within the spatial frequency bandwidth 0.2-20 cycles/m.

In the 2-D wave-number domain, the power spectrum of the rough surface is specified by

$$P_h(\mathbf{k}) = 4 \pi \nu H^2 |\mathbf{Q}|^{-1/2} [u^2(\mathbf{k}) + 1]^{-(\nu+1)},$$
 (1)

where  $\nu$  is related to the fractal dimension D via

$$\nu = 3 - D,\tag{2}$$

u is the dimensionless norm of the wave number k, given by

$$u(\mathbf{k}) = \sqrt{(k/k_s)^2 \cos^2(\zeta - \zeta_s) + (k/k_n)^2 \sin^2(\zeta - \zeta_s)}, \quad (3)$$

and Q is the scale matrix, related to the characteristic wave numbers  $k_n$  and  $k_s$  by

$$\mathbf{Q} = k_n^2 \mathbf{e}_n \mathbf{e}_n^T + k_s^2 \mathbf{e}_s \mathbf{e}_s^T. \tag{4}$$

O provides a scaling mechanism which controls the relative amount of energy present in the surface at a given azimuth and frequency. From (1) it is observed that at wave numbers much less than  $k_n$ , the power spectrum will be essentially flat, placing an upper bound on the low-frequency energy content of the interface roughness. For very large wave numbers, such that  $u(\mathbf{k}) \gg 1$ , the rate of spectral rolloff in the k direction is governed by a pole of order  $2(\nu+1)$ , the location of which is determined by the characteristic, or corner, wave numbers of the surface spectrum. The introduction of a pole at the characteristic wave number leads to a piecewise-linear power-law dependence, thereby permitting the representation of naturally occurring bottom structure while guaranteeing finite moments of the large-scale surface structure. It should be noted, however, that this attribute of the model necessarily assumes that large-scale trends in the actual bathymetry are known a priori and corrections to the local angle of incidence made in the data.

The capability of the model to simulate naturally occurring bottom microroughness is illustrated in Fig. 1. Figure 1(a) depicts a smooth, rippled sediment field, characteristic of the influence of abyssal currents on sediment distribution. This surface realization is a member of an ensemble characterized by the parameter set H=0.0125 m,  $k_n=2.5$  cycles/m,  $k_s=0.5$  cycles/m, D=1.6, and  $\zeta_s=45^{\circ}$ , measured clockwise from the y axis. Figure 1(b) depicts a rough, isotropic province characteristic of an accumulation of shell fragments. This surface family is characterized by the parameter set H=0.0042 m,  $k_n=k_s=1.5$  cycles/m, and D=2.53, representative of one of the isotropic, continental shelf microroughness sites studied recently by Briggs. 8

#### II. THE ACOUSTIC SCATTERING MODEL

The model for bottom reverberation employed in this work is based on the 3-D Kirchhoff approximation to the Helmholtz integral equation. The approximation is generally considered valid if the surface appears locally flat relative to the incident wavelength. In a recent empirical study of scattering from randomly rough surfaces, Thorsos<sup>9</sup> established a quantitative criterion for Kirchhoff validity for backscattering near normal incidence to be approximately  $l/\lambda \ge 1$ , where l is the surface correlation length, and  $\lambda$  is the incident wavelength. Further, to preclude the possibility of backscattering enhancement due to multiple scattering effects, an upper bound is placed on surface rms height such that  $H/\lambda \le 0.25$ .

As the form of the optimum detector is derived from the joint pdf of the reverberation process, an analytical representation for the covariance function of the bottom reverberation in terms of the microroughness covariance function is required. In this section, an analytical expression for the scattered field spatial covariance for an arbitrary surface rough-

ness spectrum, due to Restrepo and McDaniel,<sup>5</sup> is merged with the Goff-Jordan roughness spectrum to obtain the desired result.

The geometry for the rough interface scattering problem is given in Fig. 2. From Beckmann and Spizzichino, <sup>10</sup> the Kirchhoff approximation for the field scattered from a randomly rough interface is given by

$$p(\mathbf{r}) = \frac{ike^{ik(r'+r_1)}}{2\pi r'r_1} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} D(x,y) \Re(x,y) (\alpha \xi_x + \beta \xi_y + \gamma)$$

$$\times \exp[ik(\alpha x + \beta y - \gamma \xi)]dx dy, \tag{5}$$

where  $\alpha$ ,  $\beta$ , and  $\gamma$  represent the x, y, and z components of the unit difference between the incident and scattered wave vectors, D is a Gaussian illumination function introduced to suppress edge effects, and  $\Re$  is the local plane-wave reflection coefficient, dependent on the densities and sound velocities associated with the media that comprise the interface, as well as on the local angle of incidence. After, an integration by parts, it can be shown<sup>10</sup> that the expression for the scattered field (5) reduces to

$$p(\mathbf{r}) = \frac{ike^{ik(r'+r_1)}}{2\pi r'r_1} f(\theta) \int_{\infty}^{\infty} \int_{\infty}^{\infty} D(x,y) \Re(x,y)$$
$$\times \exp[ik(\alpha x + \beta y - \gamma \xi)] dx \ dy, \tag{6}$$

where

$$f(\theta) = \frac{1 + \cos \theta_0 \cos \theta_i + \sin \theta_0 \sin \theta_i \sin \phi_i}{\cos \theta_0 (\cos \theta_0 + \cos \theta_i)}.$$
 (7)

In this work, where interface roughness is assumed to be the sole mechanism responsible for scattering,  $\Re$  is a constant equal to unity. However, it should be noted that the modeling approach is sufficiently flexible to allow for the treatment of a finite impedance, anelastic boundary with an appropriate representation of  $\Re(x,y)$ .

Restrepo and McDaniel have shown<sup>5</sup> that the spatial covariance of the scattered field,  $\langle p(\mathbf{r}_1)p^*(\mathbf{r}_2)\rangle$ , is related to the exponential of the surface covariance function via the Fourier transform relationship

$$\Gamma(\mathbf{K}) = \frac{Ak^{2}f_{1}(\theta)f_{2}(\theta)e^{ik(r_{1}-r_{2})}}{8\pi^{2}r'^{2}r_{1}r_{2}}$$

$$\times e^{-\Gamma}e^{-(k^{2}/8)(\alpha^{2}L^{2}+\beta^{2}L^{2})}$$

$$\times \int_{-\infty}^{\infty}d\mathbf{r}_{\perp} De^{ik(\mathbf{r}_{\perp}\cdot\mathbf{K})}e^{GC(\mathbf{r}_{\perp})},$$
(8)

where  $G = k^2 H^2 \gamma^2$ , **K** is the horizontal component of the unit change in wavevector, A is the area of the beam footprint, r is the range from the source to the origin of the surface patch,  $\mathbf{r}_{\perp}$  is the position vector in the horizontal lag plane, and  $C(\mathbf{r}_{\perp})$  represents the rough surface covariance function. For the details concerning the derivation of (8), the reader is referred to Ref. 4.

The receiver array geometry employed in this simulation study is shown in Fig. 3. A horizontal array is used to sample the azimuthal dependence of the scattered field covariance induced by the anisotropic bottom microroughness correla-

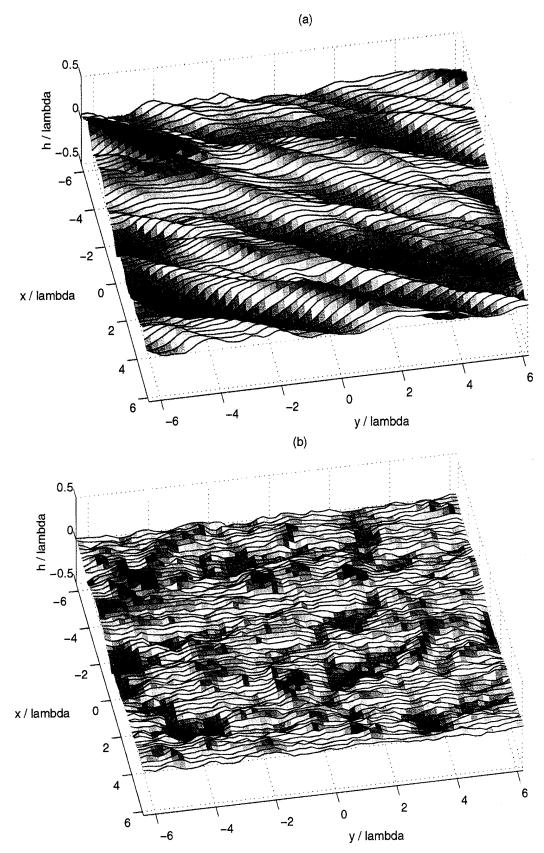


FIG. 1. Simulated realizations of seafloor relief: (a) smooth, sedimented ripple field, (b) rough, isotropic province influenced by accumulation of surficial shell hash.

tion structure. The dimensions associated with the array geometry, e.g., beamprint area, range to insonified seafloor patch, maximum roughness correlation length, etc., were selected to satisfy the far-field criterion given by  $^{11}$  (1)  $L \ll r$ , (2)  $lL/\lambda \ll r$ , where L is the diameter of the insonified path, l is the maximum roughness correlation length, r is the range from the source to the origin of the surface path, and  $\lambda$  is the acoustic source wavelength. The size of the insonified sur-

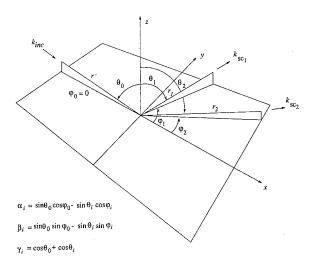


FIG. 2. Scattered geometry.

face patch was selected to span a large number of surface correlation lengths, generally between 10 and 50, depending on the surface parameters. The selection of system parameters such as array length and element spacing (normalized with respect to acoustic wavelength) was largely influenced by the size of the insonified surface patch. For example, increasing the patch size has the effect of decreasing the correlation length of the reverberation field, thereby reducing the element spacing needed to adequately sample the characteristic decay scale of the bottom reverberation spatial coherence. In the simulation study, the maximum dimension of the insonified surface patch was driven by a trade-off between the need to adequately represent the large correlation length features of the surface roughness and computational considerations in the numerical evaluation of (8). The receiver element spacing was governed by the need to sample the scattered field spatial coherence as densely as possible without oversampling, which can lead to an ill-conditioned reverberation covariance matrix. An acceptable receiver element spacing was determined to be equal to  $l_0/2$ , where  $l_0$  is the correlation length of the scattered field measured along the receiver array. The beamwidth specified by the Gaussian taper function was selected to insure that a region spanning a minimum of one surface correlation patch was uniformly insonified to within 3 dB. For the array geometry employed in this study, this requirement translated into an effective half-power beamwidth of 0.3°. It should also be pointed out that the  $l_0/2$  sampling scheme outlined above allows for enough space between adjacent receiver elements such that each element in the array may be composed of a smaller sub-array, capable of high-resolution beamforming for the rejection of any undesirable specular component present in the reverberation signature.

The sensitivity of the scattered field spatial covariance function to Goff-Jordan type microroughness correlation structure is illustrated in Fig. 4 for the surface families depicted in Fig. 1. Figure 4(a) depicts the preferential steering of the bottom interacting sound in the direction normal to the ripples in the sediment field (in the direction of positive y), while the banded structure of the scattered field covariance in Fig. 4(b) reflects the isotropic nature of the shell hash sediment field.

#### III. THE OPTIMUM DETECTOR

#### A. A general framework

The objective of the optimum detector is to make the best decision as to the presence or absence of an unknown object suspended in the water column in the vicinity of an uncertain seafloor based on a single observation of the backscattered acoustic signature. The problem is thus cast as a hypothesis test between  $H_0$ , under which the observation consists of bottom reverberation and noise alone, and  $H_1$ , in which the target signature is also present in the observation. It is well known that the optimum solution to the detection problem is for the signal processing algorithm to implement the likelihood ratio followed by a threshold whose value is determined by a particular decision criterion (e.g., minimum probability of error, Neyman-Pearson, etc.). 12 The likelihood ratio,  $\lambda(\mathbf{r})$ , is defined in terms of the ratio of the probability densities of the observation under each hypothesis

$$\lambda(\mathbf{r}) = \frac{p(\mathbf{r}|H_1)}{p(\mathbf{r}|H_0)}.\tag{9}$$

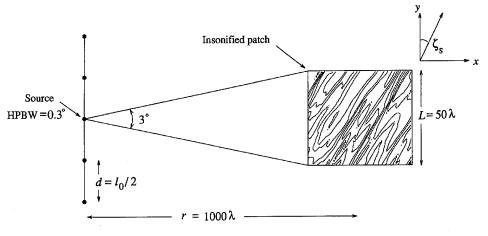


FIG. 3. Receiver array geometry (plan view).

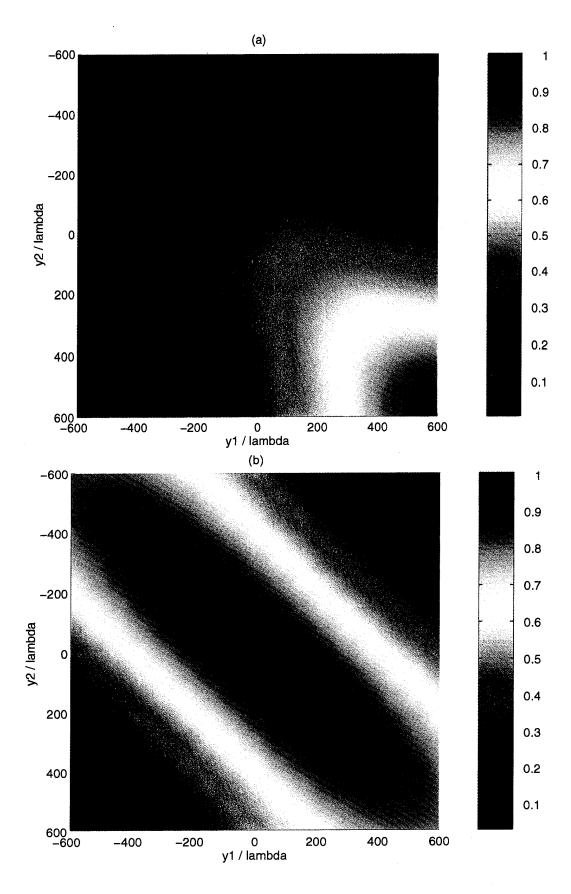


FIG. 4. Scattered field spatial covariance functions corresponding to microroughness classes depicted in Fig. 1: (a) rippled sediment, (b) isotropic shell fragments.

The observation vector consists of a single snapshot from an N-element horizontal receiver array. The observation under each hypothesis is defined as

$$H_0: \mathbf{r} = \mathbf{w} + \mathbf{b}(\mathbf{\Phi}, \mathbf{\Psi}),$$
  
 $H_1: \mathbf{r} = \mathbf{w} + \mathbf{b}(\mathbf{\Phi}, \mathbf{\Psi}) + \mathbf{h}(\mathbf{S}, \boldsymbol{\beta}, \mathbf{\Phi}),$ 
(10)

where w represents additive white Gaussian noise, b is the complex reverberation field, and h is the complex target signature. Both the reverberation and target signatures are modeled as random vectors dependent upon source, target, and environmental parameters. The object's position in the water column is denoted by S. The properties of the unknown object (e.g., size, shape, material composition) are represented by the amplitude and phase of its complex acoustic reflection coefficient, and are denoted by  $\beta$ . The transmitted signal parameters (e.g., amplitude, phase, center frequency, beamwidth) are generally assumed to be known and are denoted by  $\Phi$ . The statistical parameters associated with the Goff-Jordan model for the anisotropic bottom microroughness are denoted by  $\Psi$ . This treatment assumes that any secondary reverberation, i.e., interaction of the target signature with the rough bottom, is negligible in comparison with the direct backscattered bottom reverberation.

A generalized theoretical framework for deriving the optimum detector in an uncertain reverberation-limited environment is first presented. It is shown that the optimum detector may be derived without precise knowledge of the functional form of the underlying observation pdf by describing the random fluctuations of the complex reverberation field in terms of the physics of wave scattering. The treatment utilizes full field (i.e., amplitude and phase) modeling of the bottom interacting acoustic field given by (5). The approach is based on the premise that the reverberation pdf is inherently represented in the scattering physics through the ensemble of scattered field realizations associated with a particular environmental parameter set. As a result, the physics-based approach provides a mechanism for automatically incorporating the scattered field pdf into the detector formulation, whether or not it is known exactly. For the special case of Gaussian reverberation, it is then demonstrated that a computationally efficient implementation of the optimum detector is possible which makes explicit use of the analytical representation for the scattered field spatial coherence given by (8).

The starting point for the derivation of the optimum detector is the pdf of the observation  $\mathbf{r}$  conditioned on the environmental parameter set  $\mathbf{\Psi}$ . Assuming the noise and bottom reverberation to be statistically independent random processes, it is straightforward to show that the conditional pdf,  $p_{\mathbf{r}|\mathbf{\Psi}}(\mathbf{r}|\mathbf{\Psi})$ , may be expressed in terms of a convolution between the noise and reverberation pdf's:

$$p_{\mathbf{r}|\Psi}(\mathbf{r}|\Psi) = \int_{\mathbf{b}} p_{\mathbf{w}}(\mathbf{r} - \mathbf{b}(\Psi)) p_{\mathbf{b}|\Psi}(\mathbf{b}|\Psi) d\mathbf{b}. \tag{11}$$

It is assumed that prior knowledge of the target parameters  $\beta$ , target location S, and bottom roughness parameters  $\Psi$ , is uncertain and specified only in terms of probability distributions. Therefore, the marginal pdf of the observation must be

obtained from the joint density of  $\mathbf{r}$  and the acoustic environmental parameter space  $\{\beta, \mathbf{S}, \mathbf{\Psi}\}$  by integrating as follows:

$$p_{\mathbf{r}}(\mathbf{r}) = \int_{\beta} \int_{\mathbf{S}} \int_{\mathbf{\Psi}} p_{\mathbf{r},\beta,\mathbf{S},\mathbf{\Psi}}(\mathbf{r},\beta,\mathbf{S},\mathbf{\Psi}) d\mathbf{\Psi} d\mathbf{S} d\beta.$$
 (12)

Applying Bayes' rule and using (11), the observation pdf may be rewritten as

$$p_{\mathbf{r}}(\mathbf{r}) = \int_{\beta} \int_{\mathbf{S}} \int_{\mathbf{\Psi}} \int_{\mathbf{b}} p_{\mathbf{w}}(\mathbf{r} - \mathbf{b}(\mathbf{\Psi})) p_{\mathbf{b}|\mathbf{\Psi}}(\mathbf{b}|\mathbf{\Psi}) p(\beta)$$
$$\times p(\mathbf{S}) p(\mathbf{\Psi}) d\mathbf{b} d\mathbf{\Psi} d\mathbf{S} d\beta, \tag{13}$$

where  $p(\beta)$ , p(S), and  $p(\Psi)$  represent the *a priori* probability densities associated with the target parameters, target position, and bottom roughness spectral parameters, respectively. In the limit of maximum uncertainty, these densities may be defined as uniform over some predetermined range of parameter values. (The Bayesian formulation does require prior specification of bounds on the space of possible parameter values.) More specific prior information may be incorporated as the knowledge of the acoustic environment is updated. For example, in the case of  $\Psi$ , techniques are available for the estimation of bottom roughness spectral parameters from the same full-field acoustic data used by the detection algorithm.<sup>14</sup>

In the general case, all of the integrations in (13) may be performed numerically without difficulty. Of particular importance is the integration with respect to  $\mathbf{b}(\mathbf{\Psi})$ , the space of backscattered acoustic field realizations. With an accurate physical model for acoustic scattering, it is possible to approximate this integral for an arbitrary or unknown pdf via the technique of Monte Carlo integration. Briefly stated, Monte Carlo integration approximates the pdf  $p_{\mathbf{b}|\Psi}(\mathbf{b}|\Psi)$  by sampling the space of reverberation field realizations according to the probability distribution prescribed by the physical scattering model. In this respect, full-field acoustic modeling enables the optimum detector to be realized without knowledge of the analytical form of the reverberation pdf. It is necessary to point out that this flexibility is balanced by the fact that the Monte Carlo integration technique can be computationally expensive depending on the number of independent environmental realizations required to achieve a stable integration result. The investigation of the Monte Carlo detector implementation for an arbitrary (both Gaussian and non-Gaussian) reverberation pdf will be the subject of a future paper.

From (13), it is clear that any uncertainty with respect to prior knowledge of the acoustic environment, or, more specifically, the seafloor microroughness spectrum, will affect the form of the underlying observation pdf. As the uncertainty associated with the reverberation pdf is present under both hypotheses, this "detection in uncertain reverberation"

problem is an example of a "doubly composite" hypothesis problem. In the doubly composite problem, the integration of the joint pdf over the imperfectly known environmental parameter space, in this case denoted by  $\Psi$ , must be performed under each hypothesis *prior* to forming the likelihood ratio. Assuming perfect knowledge of the transmitted signal parameters  $\Phi$ , the likelihood ratio is thus given by

$$\lambda(\mathbf{r}) = \frac{\int_{\mathbf{\Psi}} \int_{\mathbf{S}} \int_{\beta} p_{\mathbf{r},\beta,\mathbf{S},\mathbf{\Psi}|H_{1}}(\mathbf{r},\beta,\mathbf{S},\mathbf{\Psi}|H_{1}) d\beta \ d\mathbf{S} \ d\mathbf{\Psi}}{\int_{\mathbf{\Psi}} p_{\mathbf{r},\mathbf{\Psi}|H_{0}}(\mathbf{r},\mathbf{\Psi}|H_{0}) d\mathbf{\Psi}}.$$
(14)

Substituting (13) into (14) under each hypothesis, the likelihood ratio becomes

$$\lambda(\mathbf{r}) = \frac{\int_{\beta} \int_{\mathbf{S}} \int_{\Psi} \int_{\mathbf{b}} p_{\mathbf{w}} [\mathbf{r} - \mathbf{b}(\boldsymbol{\Psi}) - \mathbf{h}(\mathbf{S}, \boldsymbol{\beta}, \boldsymbol{\Phi})] p_{\mathbf{b}|\boldsymbol{\Psi}} (\mathbf{b}|\boldsymbol{\Psi}) p(\boldsymbol{\beta}) p(\mathbf{S}) p(\boldsymbol{\Psi}) d\mathbf{b} d\boldsymbol{\Psi} d\mathbf{S} d\boldsymbol{\beta}}{\int_{\Psi} \int_{\mathbf{b}} p_{\mathbf{w}} (\mathbf{r} - \mathbf{b}(\boldsymbol{\Psi})) p_{\mathbf{b}|\boldsymbol{\Psi}} (\mathbf{b}|\boldsymbol{\Psi}) p(\boldsymbol{\Psi}) d\mathbf{b} d\boldsymbol{\Psi}}.$$
(15)

Equation (15) represents the general form of the optimum detector in an uncertain reverberation-limited environment. The Bayesian framework provides the mechanism for introducing a probabilistic description of the state of prior knowledge concerning the acoustic environment. In essence, the Bayesian formulation prescribes the optimum weighting scheme for summing the contributions of each member of the space of possible environments to the likelihood ratio. The fact that the likelihood ratio is inherently linked to the physical model for acoustic scattering means that the detector is able to exploit the well-understood physics of the scattering process to optimally filter out the randomly fluctuating, signal-masking component of the observation.

#### B. A special case: Gaussian reverberation

The special case of Gaussian reverberation is now considered. Both the noise and reverberation are assumed to be statistically independent N-dimensional, Gaussian random vectors characterized by covariance matrices  $\Lambda_w$  and  $\Lambda_b$ , respectively. In this case, the convolution of the noise and reverberation pdf's may be performed analytically. It is straightforward to show that the resulting pdf is also Gaussian, with the conditional density of the observation specified by

$$p_{\mathbf{r}|\boldsymbol{\Psi}}(\mathbf{r}|\boldsymbol{\Psi}) = [(2\pi)^{N/2}|\mathbf{Q}(\boldsymbol{\Psi})|^{1/2}]^{-1} \times \exp[-\frac{1}{2}\mathbf{r}^*\mathbf{Q}^{-1}(\boldsymbol{\Psi})\mathbf{r}], \tag{16}$$

where  $\mathbf{Q}(\mathbf{\Psi}) = \mathbf{\Lambda}_{\mathbf{w}} + \mathbf{\Lambda}_{\mathbf{b}}(\mathbf{\Psi})$ . The marginal pdf for the observation is obtained from the joint density  $p_{\mathbf{r},\beta,\mathbf{S},\mathbf{\Psi}}(\mathbf{r},\beta,\mathbf{S},\mathbf{\Psi})$  following the same procedure as in the general case, except that (16) is substituted in place of the convolution integral in (13).

In order to derive the final form of the likelihood ratio in this special case, it will be helpful to introduce a probabilistic model for the uncertain target signature. The acoustic target transfer function,  $h_n$ , observed at a particular receiver element, includes both a random and deterministic component and is given by

$$h_n(\mathbf{S}, \boldsymbol{\beta}, \boldsymbol{\Phi}) = A a_n e^{j\Theta + \phi_n}. \tag{17}$$

The deterministic component,  $a_n e^{j\phi_n}$ , is associated with the propagation of the acoustic return through the water column. For the purpose of this investigation, the refraction index is

assumed to be perfectly known and constant. The random component,  $Ae^{j\Theta}$ , samples the object's complex wavenumber spectrum. The object's amplitude reflection coefficient, A, is a function of its size and is defined as

$$A = a_0 \sigma_A \,, \tag{18}$$

where  $a_0$  is a sample from a Rayleigh distribution with parameter  $\sigma$  equal to 1, and  $\sigma_A$  represents a scale factor related to the target strength. The phase of the reflection coefficient,  $\Theta$ , follows a uniform distribution on the interval  $[-\pi,\pi]$ , and is a function of the objects material composition and  $O(\lambda)$  perturbations to the object's position in the water column. As the focus of this paper is on accurate modeling of the bottom microroughness rather than on the physics of object scattering, this formulation permits uncertainty with respect to object size, shape, and material composition to be incorporated in an analytically tractable, intuitively meaningful way in the absence of a physical model for the scatterer.

The physical connection between the scale factor  $\sigma_A$  and object size may be illustrated with a simple example. First, it is helpful to define the signal-to-reverberation ratio per receiver element, SRR, as follows:

$$SRR = \frac{K\sigma_A^2}{\text{tr}(\mathbf{Q})},\tag{19}$$

where K is the number of receiver elements and  $\operatorname{tr}(\cdot)$  denotes the matrix trace. For the receiver array geometry and microroughness classes of interest in this study (i.e., for which the first-order Kirchhoff approximation is satisfied), the SRR was empirically found to lie in the range -5 to -15 dB. Recall that Urick<sup>15</sup> defines the target strength of a rigid finite cylinder, in the limit  $ka \gg 1$ , as

$$\sigma_A^2 = al^2/2\lambda. \tag{20}$$

At an acoustic source frequency of 12 kHz, using (20) it is straightforward to show that an SRR=-10 dB corresponds approximately to a rigid cylinder of radius 6.4 cm and length 6.4 cm.

Assuming the amplitude and phase of the target reflection coefficient to be independent, the a priori pdf of the target parameters,  $p(\beta)$ , can be expressed as the product

$$p(\beta) = p(A)p(\Theta), \tag{21}$$

where p(A) is Rayleigh and  $p(\Theta)$  is uniform. Using (21), it can be shown that if the transmitted signal is narrow band, the integration over the object parameter space  $\beta$  in (15) may be performed analytically 16 to yield

$$\lambda(\mathbf{r}) = \frac{\int_{\mathbf{\Psi}} \int_{\mathbf{S}} |\mathbf{Q}(\mathbf{\Psi})|^{-1/2} (B+E+1)^{-1} \exp[|R|^2 / (B+E+1)] p(\mathbf{S}) p(\mathbf{\Psi}) d\mathbf{S} d\mathbf{\Psi}}{\int_{\mathbf{\Psi}} |\mathbf{Q}(\mathbf{\Psi})|^{-1/2} \exp(-a_0^2 B/2) p(\mathbf{\Psi}) d\mathbf{\Psi}}$$
(22)

Notice that the detector implementation based on the Gaussian reverberation assumption precludes the need for Monte Carlo integration over the space of scattered field realizations, leaving only numerical integration over the parameters **S** and  $\Psi$ . The quantities B, E, and R are functions of **S** and Ψ given by

$$B = \mathbf{r}^* \mathbf{Q}^{-1} \mathbf{r} / a_0^2, \quad E = \mathbf{h}^* \mathbf{Q}^{-1} \mathbf{h} / 2,$$

$$R = \mathbf{h}^* \mathbf{Q}^{-1} \mathbf{r} / 2, \tag{23}$$

where  $a_0$  is the normalized Rayleigh variate associated with the target amplitude reflection coefficient, E is a measure of the self-energy in the target signature, R is a measure of the correlation of the target signature with the observation vector, and B indicates the extent to which the covariance matrix  $\Lambda_{\rm h}$  correctly characterizes the measured reverberation. In effect, B is a measure of the residual error incurred in projecting the observation **r** onto the eigenvector space of  $\Lambda_b$ . If the model assumed for the bottom roughness is incorrect, it follows that  $\mathbf{r}$  is not a member of the ensemble of backscattered pressure field vectors characterized by  $\Lambda_{\rm b}$ . In such case, B will be very large and the probability associated with the observation vector under each hypothesis will be extremely small. The result will be insignificant separation between the pdf's of the observation (or likelihood ratio) under each hypothesis and poor detection performance, reflected by an ROC curve lying very near to the "chance"

diagonal. In the next section, ROC curves are used to quantify the impact that inaccurate modeling of the reverberation spatial coherence can have on detection performance.

To aid in the interpretation of the optimum detector structure for the Gaussian reverberation case, a block diagram of the processor is given in Fig. 5.

#### IV. ROC SIMULATION RESULTS

In this section, the performance of the likelihood ratio detector for the case of Gaussian reverberation is considered. First, the sensitivity of the likelihood ratio detector designed for a known environment to incorrect prior knowledge of the seafloor roughness spectrum is examined. It is then shown that the optimum detector defined by (22) provides a robust means of treating the uncertainty regarding the wave-number spectrum of the interface roughness. Detection performance is characterized in terms of receiver operating characteristic (ROC) curves. The ROC curve, defined as the probability of detection versus the probability of false alarm, is the standard quantitative measure for assessing detection performance.

A few brief comments about the ROC curves. In all cases, ROC curves were computed from 500 independent realizations of the observation vector under each hypothesis. Individual observations of the reverberation field were simu-

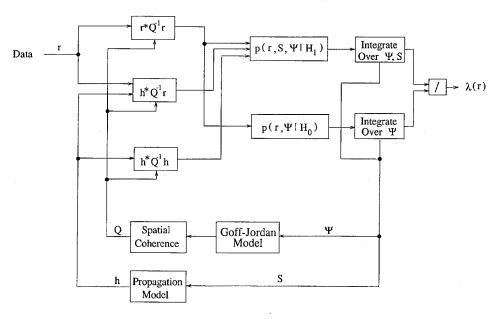


FIG. 5. Bayesian optimum detector block diagram.

lated by applying Eq. (6) to independent realizations of Goff-Jordan microrelief. ROCs grouped within a given figure were obtained using the same reverberation data set. Individual ROCs within a figure are then labeled in accordance with state of prior knowledge assumed by the processor. The ROCs are plotted on normal-normal coordinate axes to provide increased accuracy near the endpoints of each ROC curve. The dotted reference template that appears in each figure represents a family of analytically determined constant detection factor (DF) ROC curves corresponding to the SKE case. While the SKE case was not explicitly considered in this study (recall the Rayleigh/uniform assumption assigned to the unknown target signature), the SKE template provides a convenient visual reference for characterizing detection performance. The reference curves are spaced nonlinearly with respect to DF, corresponding to DFs of 0 (chance diagonal), 1, 4, 9, 16 dB, etc., moving northwest from the chance diagonal. For clarity of presentation, the SRR of the data was generally chosen such that the matched case ROC yielded an SKE equivalent DF of approximately 12 dB. Stated another way, the SRR was chosen such that a detection probability of 0.9 corresponded to a false alarm level of 0.01. In all cases the dominant source of interference is reverberation, with a level approximately 40 dB above that of the ambient noise.

In this work, the matched case, defined as the situation where the prior knowledge assumed by the processor exactly describes the reverberation environment, establishes the upper bound on the detection performance for a given data set.

#### A. Cost of ignoring reverberation coherence

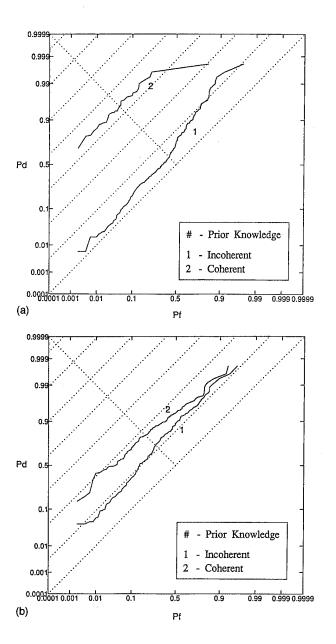
To establish the importance of incorporating scattered field spatial coherence information into the design of the likelihood ratio detector, the cost of ignoring reverberation coherence when it is present in the data is first considered. This is done by comparing the performance of two different processors, one which is based on exact prior knowledge of the reverberation covariance matrix, and one which assumes that observations made at adjacent array elements are spatially independent. A sequence of three data sets is employed, with bottom reverberation components characterized as highly coherent, moderately coherent, and incoherent, respectively. The corresponding sets of ROC curves are depicted in Fig. 6. In this example, the degree of coherence present in the backscattered acoustic field is varied by changing the spacing between array elements, d, maintaining a fixed beamprint area. For all three sets of ROCs, the SRR as defined in the previous section is approximately equal to -11 dB. The sequence demonstrates the performance degradation incurred by the likelihood detector in failing to account for the scattered field spatial coherence induced by the correlation structure of the bottom roughness.

Figure 6(a) demonstrates the ROC performance for the case of small element spacing d, where the coherent structure of the bottom interacting sound field is most densely sampled. Observe that the processor which correctly assumes coherence to be present a priori significantly outperforms that which is based on the independence assumption. As the sensor spacing is increased, from  $d=l_0/2$  to  $d=l_0$  [Fig. 6(b)], and finally to  $d=2l_0$  [Fig. 6(c)], the correlation between adjacent sensors decreases until the independence assumption is essentially satisfied. This fact is reflected in the convergence of the two processors' respective ROCs at approximately  $d/l_0=2$ , corresponding to the array spacing for which the correlation between adjacent sensors has decayed to zero. Notice that the convergence occurs as a result of movement on the part of both ROCs, i.e., downward drift for the processor associated with the coherent prior assumption and upward drift for that associated with the independence assumption. This effect is summarized in Fig. 7 which depicts detection factor as a function of sensor spacing. The figure demonstrates that the best case DF observed when coherence is present in the data and correctly accounted for in the signal processing (upper curve) greatly exceeds the highest DF attainable when the independence assumption is satisfied. The magnitude of the performance difference may have significant implications for conventional array design strategies which generally seek to prewhiten the observation in an effort to conform to the requirements of canonical signal processing methodologies. It is also noted that a similar effect is observed when coherence is controlled by changing the size of the beamprint for a fixed element spacing (increased beamprint area corresponding to decreased coherence).

#### B. Sensitivity to incorrect priors and optimum detector performance in the presence of uncertainty

The sensitivity of the likelihood ratio designed for a known environment to inaccurate prior knowledge of the reverberation spatial coherence, and the capacity of the optimum detector formulation in (22) to treat the problem of environmental uncertainty are now examined. Since a horizontal array will be primarily sensitive to azimuthal variation in the roughness wave-number spectrum, only mismatch with respect to aspect ratio and lineation direction is considered. In this simulation example, the actual seafloor roughness spectrum is characterized by the parameter set H = 0.0125 m,  $k_n = 2.5$  cycles/m,  $k_s = 0.5$  cycles/m, D=1.6, and  $\zeta_s=30^\circ$ . Maintaining fixed H and D, the four mismatch cases depicted in Fig. 8 are defined by: (a) isotropic with  $k_n = k_s = 2.5$  cycles/m; (b) a = 2.5 and  $\zeta_s = 45^\circ$ ; (c) a=2.5 and  $\zeta_s=15^\circ$ ; and (d) a=10 and  $\zeta_s=30^\circ$ . In each case, curve 1 denotes the performance of the mismatched processor, while curve 2 denotes the performance of the matched case. The ROC results clearly demonstrate the performance impact of an incorrect prior assumption concerning the reverberation coherence. For example, using the matched case as a reference, consider the ROC performance corresponding to a false alarm level of 0.01 and a detection probability of 0.9. For the sampling of environmental mismatch cases represented in Fig. 8, the detection probability for a fixed false alarm rate of 0.01 suffers a minimum drop off to 0.5 in the best case scenario [Figure 8(c)], and a maximum dropoff to 0.01 in the worst case scenario [Figure 8(d)].

One possible way to treat the problem of mismatch or uncertainty with respect to reverberation coherence structure would be to prewhiten the observation. However, in the previous section, it was shown that whitening the observation (in this case via array design) is tantamount to destroying



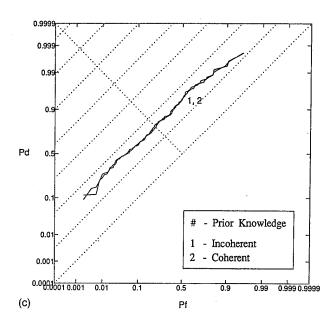


FIG. 6. Cost of ignoring reverberation coherence: (a) highly coherent data, (b) moderately coherent data, and (c) incoherent data.

information that can be used to filter out the random signalmasking portion of the acoustic return. While the sensitivity to incorrect prior knowledge of the roughness spectrum would be removed, it would only be accomplished at the expense of a considerable reduction in maximum possible detection performance (see Fig. 7).

The capacity of the optimum detector in (22) to provide robust detection performance in the presence of inexact prior knowledge without compromising the maximum possible detection performance is demonstrated in Fig. 9. In this example, the space of possible environments  $\Psi$  over which integration is performed in (22) is composed of a total of 15 surface classes, obtained by permuting the following parameter sets:  $a = \{2.5,5\}$  and  $\zeta_s = \{0^{\circ},15^{\circ},30^{\circ},45^{\circ}\}$ ,  $a = \{7.5,10\}$  and  $\zeta_s = \{0^{\circ},15^{\circ},30^{\circ}\}$ , plus an isotropic class characterized by  $k_n = k_s = 2.5$  cycles/m. Prior knowledge of the seafloor roughness spectrum, as represented by  $p(\Psi)$ , is defined by a uniform pdf spanning the space  $\Psi$ . As before, curve 2 denotes the matched result, while curve 1 now de-

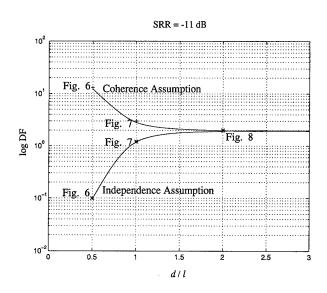


FIG. 7. Cost of ignoring reverberation coherence. Detection factor versus element spacing.

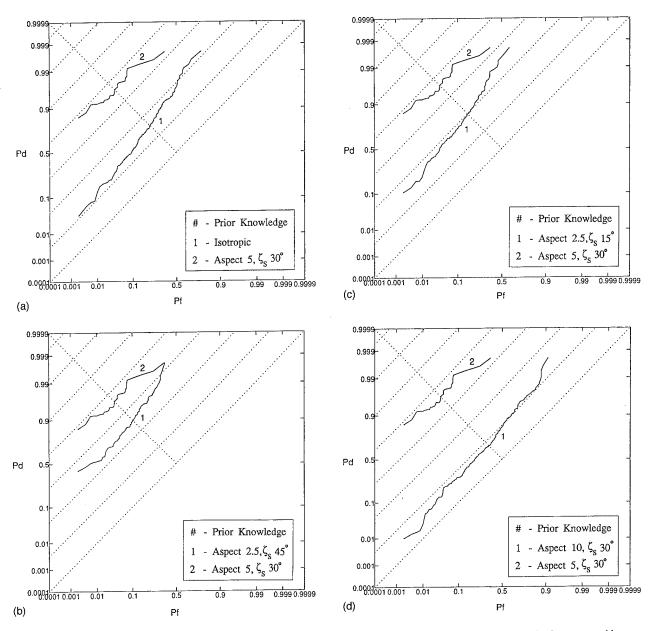


FIG. 8. Sensitivity of likelihood ratio detector to mismatch. In all cases, the actual bottom is characterized by a=5,  $\zeta_s=30^\circ$ . Seafloor assumed by processor a priori given by: (a) isotropic, (b) a=2.5,  $\zeta_s=45^\circ$ , (c) a=2.5,  $\zeta_s=15^\circ$ , and (d) a=10,  $\zeta_s=30^\circ$ .

notes the optimum result. Observe that the performance of the optimum detector closely approaches the upper bound defined by the matched case ROC. This result indicates that the optimum solution is capable of sifting through the environmental uncertainty and providing robust detection performance without compromising the maximum possible performance (attainable in the event of exact prior knowledge of the reverberation coherence).

Lastly, detection performance is illustrated for the case where the ambient noise level is no longer negligible compared with the reverberation. In Fig. 10, the noise level in the data has been increased to a point approximately 20 dB below that of the reverberation, i.e.,

$$10 \log \frac{\sum_{i} \sigma_{b_{i}}^{2}}{K\sigma_{w}^{2}} = 20 \text{ dB}, \tag{24}$$

where K is the number of sensors. A degradation in performance is observed as the ROCs for both the matched and optimum processors drift toward the chance diagonal. This result reflects the impact of the increasing diagonal dominance of the covariance matrix, due to the influence of spatially white isotropic noise, on each detector's ability to exploit the knowledge of the scattering physics to recognize and filter out the reverberation interference. However, the performance of the optimum detector continues to closely track the upper bound indicated by the matched case ROC.

#### V. CONCLUSIONS

The problem of high-frequency target detection in an imperfectly known, reverberation-limited environment has been examined within a Bayesian decision-theoretic frame-

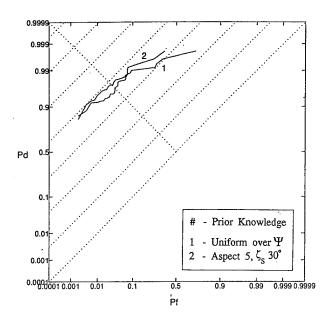


FIG. 9. Bayesian optimum detector performance.

work. The sensitivity of the likelihood ratio detector for a known acoustic environment to inexact prior knowledge of the seafloor roughness spectrum was established. Specifically, it was demonstrated that there is significant performance cost associated with discarding reverberation coherence information present in the backscattered acoustic signature. Consequently, it is believed that array design strategies may benefit considerably from sampling the coherence of the bottom interacting return instead of prewhitening the sensor measurements to conform to the data independence requirements of canonical signal processing methods. ROC curves were employed to quantify the performance degradation caused by incorrect prior assumptions concerning the acoustic scattering environment. The general form for the likelihood ratio detector in an uncertain reverberation-limited environment was derived. It is important to note that, in its

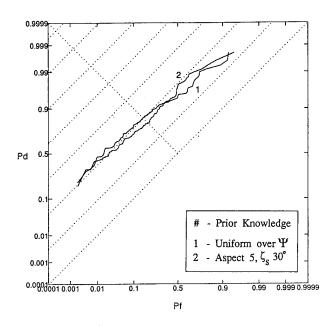


FIG. 10. Bayesian optimum detector performance (noise added).

most general form, the optimum detector may be implemented without exact knowledge of the functional form of the reverberation pdf. This suggests that the general form of the detector given by (15) could be suitable for use in a non-Gaussian reverberation environment, assuming a physical model for the acoustic scattering is available which provides access to full-field (amplitude and phase) realizations of the scattered acoustic field. It was then shown that in the case of Gaussian reverberation, a computationally efficient implementation is possible which makes explicit use of the physical model for scattered field spatial coherence due to Restrepo and McDaniel.<sup>5</sup>

It is important to emphasize that the basic approach outlined herein is not constrained to any particular physical or statistical parametrization assumed for the ocean acoustic propagation/scattering environment. While the extendibility of the Goff-Jordan parametrization to the representation of cm-scale relief has yet to be experimentally verified, the model was adopted in this work as a vehicle for the representation of randomly rough, anisotropic boundary roughness. Furthermore, it is understood that scattering from bottom roughness represents only one mechanism whereby randomness in the ocean acoustic medium is manifested. However, due to the generality of the approach, it is believed that the Bayesian decision-theoretic treatment summarized herein is extendible to any type of wave propagation/ scattering phenomenon for which a statistical physical parametrization is available, e.g., random turbulence, scattering from bubble layers, scattering from randomly inhomogeneous sediments, and internal waves.

#### **ACKNOWLEDGMENTS**

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